Contents: Active Filters

- Introduction
- Butterworth 1,2nd order Filters (Low,High)
- Narrow, Wide Band Pass Filters
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- All Pass Filter

Linear Ic Apps

21-09-2015

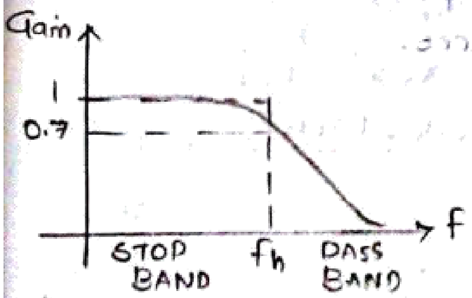
Types of filters:

(i) Passive : Active
(ii) Analog : Digital
(iii) Discrete Time : Continious Time
(iv) Time Variant : Time Invariant
(v) Infinite Impulse Response : Finite Impulse Response
(vi) Linear : Non-Linear

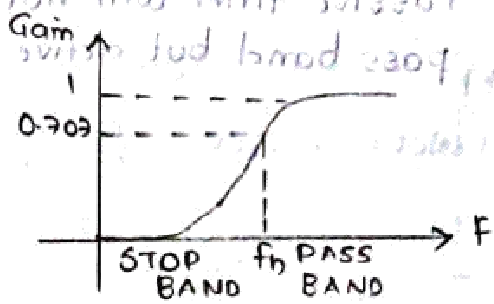
filter is a device which posses the wanted range of frequencies and opposes the unwanted range of frequencies.

Based on operation,
filters are classified as:
(i) Low pass filter
(ii) High Pass filter
(iii) Band Pass filter
(iv) Band elimination (or) Band rejection filter
(v) All pass filter

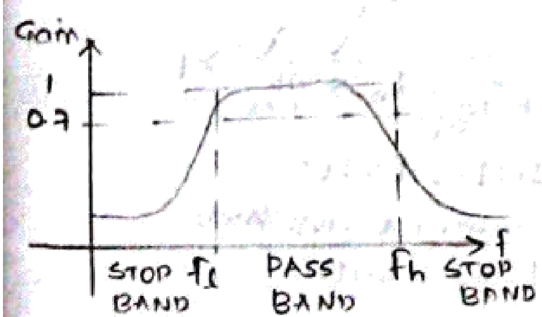
LOW PASS FILTER:



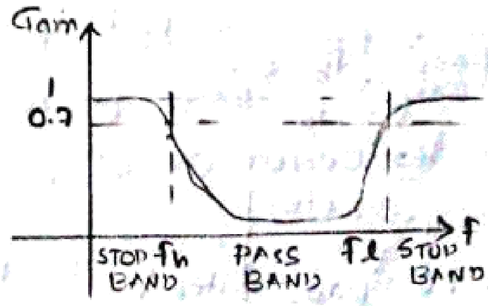
HIGH PASS FILTER:



BAND PASS FILTER:



BAND REJECT FILTER:



Cutoff frequency:

It is defined as the frequency at which the gain is:

(i) $1/\sqrt{2}$ of maximum

(ii) 70% of maximum

(iii) 3dB lesser than the maximum (in dB)

The filter maximum gain is unity.

Differences between the active filter and passive filter:

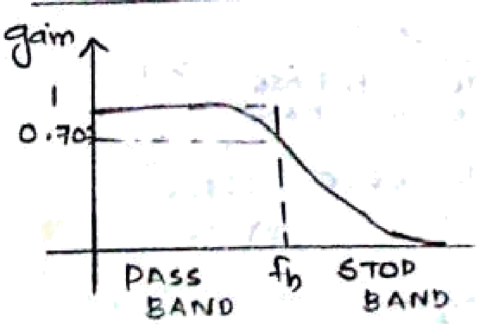
(i) A Passive filters are designed by R, L, C whereas active filters are designed by R, C and any active element like op-amp (or) transistor.

(ii) External supply is not needed in passive filter whereas it is used in active filter, because the active element has to be operated.

(iii) Roll-off (slope of the curve between pass band and stop band) is less in passive filter and it is more in active filter (ideal filter, roll-off is infinite)

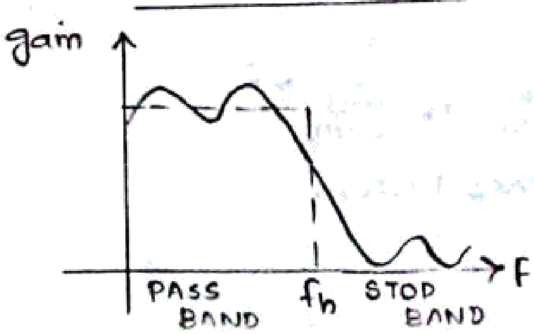
(iv) Passive filter will not produce amplification in pass band but active filter produces.

BUTTERWORTH FILTER



RIPPLES DONOT EXIST

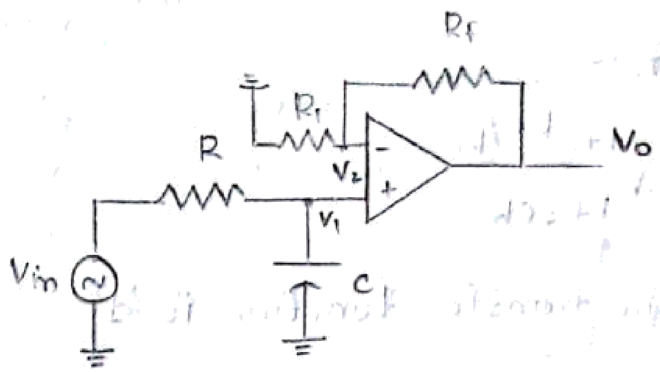
CHEBYSHEV FILTER



RIPPLES EXIST IN STOP BAND AND PASS BAND

Eg: Low pass filter

• BUTTERWORTH 1st ORDER LOW PASS FILTER (ACTIVE):



The circuit is in negative feedback.

$V_1 = V_2$.

and $V_1 = V_2 = 0$ because currents entering into op-amp input terminals is zero.

Applying the nodal analysis at non-inverting terminal of op-amp in Laplace domain,

$$\frac{V_1 - V_{in}(s)}{R} + \frac{V_1 - 0}{1/Cs} + 0 = 0$$

$$-\frac{V_{in}(s)}{R} = -Cs[V_1] - \frac{V_1}{R}$$

$$V_1[1 + sCR] = \cancel{R} \cdot \frac{V_{in}(s)}{\cancel{R}}$$

$$V_1 = \frac{V_{in}(s)}{1 + sCR}$$

Apply the nodal analysis at inverting terminal, (of op-amp)

$$\frac{V_2 - 0}{R_1} + \frac{V_2 - V_o}{R_f} + 0 = 0$$

$$\frac{V_o(s)}{1 + R_f/R_1} = V_2$$

$$V_o(s) = V_2\left[1 + R_f/R_1\right]$$

$$V_o(s) = \frac{V_{in}(s)}{1+sCR}\left[1+\frac{R_f}{R_1}\right]$$

Transfer function.

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1+R_f/R_1}{1+sCR}$$

The order of the transfer function is '1'

In frequency domain,

$$\frac{V_o(f)}{V_{in}(f)} = \frac{1+R_f/R_1}{1+j(2\pi f)RC}$$

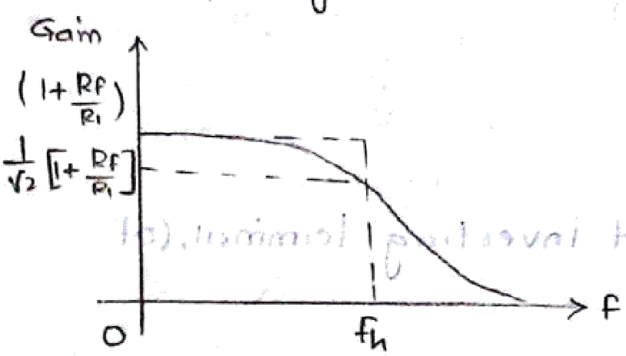
If we apply dc signal as input $(f=0)$

$$\frac{V_o}{V_{in}} = \frac{1+R_f/R_1}{1+0} = 1+\frac{R_f}{R_1} > 1$$

Input is an a.c signal with higher frequency $f=\infty$,

$$\frac{V_o}{V_{in}} = 0$$

So, frequency response,



At $f=f_h$,

$$\left|\frac{V_o}{V_{in}}\right| = \frac{1}{\sqrt{2}}\left[1+\frac{R_f}{R_1}\right]$$

So, at $f=f_h$,

$$\frac{V_o(f_h)}{V_{in}(f_h)} = \frac{1+R_f/R_1}{1+j(2\pi f_h)RC}$$

$$\left|\frac{V_o(f_h)}{V_{in}(f_h)}\right| = \frac{1+R_f/R_1}{\sqrt{1+(2\pi f_h RC)^2}}$$

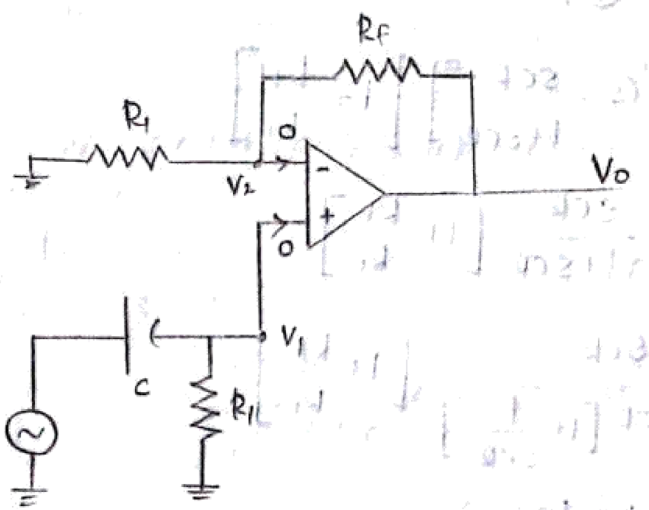
$$\frac{1}{\sqrt{2}}\left[1+\frac{R_f}{R_1}\right] = \frac{1+\cancel{R_f}/R_1}{\sqrt{1+(2\pi f_h RC)^2}}$$

$$1+2\pi f_h RC = 2$$

$$\boxed{f_h = \frac{1}{2\pi RC}}$$

22-09-15

Ist order Butterworth High Pass filter:



for negative feedback,

$$V_1 = V_2 \rightarrow (1).$$

Applying Nodal analysis at the non-inverting terminal in Laplace domain,

$$\frac{V_1 - V_{in}(s)}{1/sc} + \frac{V_1 - 0}{R} + 0 = 0$$

$$V_1\left[sC + R^{-1}\right] - V_{in}(s).sC = 0$$

$$V_1\left[\frac{sCR+1}{R}\right] - V_{in}(s).sC = 0$$

$$V_1 = V_{in}(s).\frac{sCR}{1+sCR} \rightarrow (2)$$

Applying nodal analysis at inverting terminal in laplace domain,

$$\frac{V_2 - 0}{R_1} + \frac{V_2 - V_0(s)}{R_f} + 0 = 0$$

$$V_2 \left[\frac{1}{R_1} + \frac{1}{R_f}\right] - \frac{V_0(s)}{R_f} = 0$$

$$V_2 \left[\frac{R_1 + R_f}{R_1 \cancel{R_f}}\right] = \frac{V_0(s)}{\cancel{R_f}}$$

$$V_0(s) = V_2 \left[1 + R_f / R_1\right] \rightarrow (3)$$

from (1), (2) and (3),

$$V_0(s) = \left[V_{in}(s) \cdot \frac{sCR}{1+sCR}\right]\left[1 + \frac{R_f}{R_1}\right]$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{sCR}{1+sCR}\left[1 + \frac{R_f}{R_1}\right]$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{\cancel{sCR}}{\cancel{sCR}\left[1 + \frac{1}{sCR}\right]}\left[1 + \frac{R_f}{R_1}\right]$$

$$= \frac{1 + R_f/R_1}{1 + 1/sCR}$$

Let, $A_0 = 1 + R_f / R_1$,

$$\frac{V_0(s)}{V_{in}(s)} = \frac{A_0}{1 + 1/sCR}$$

In the frequency domain,

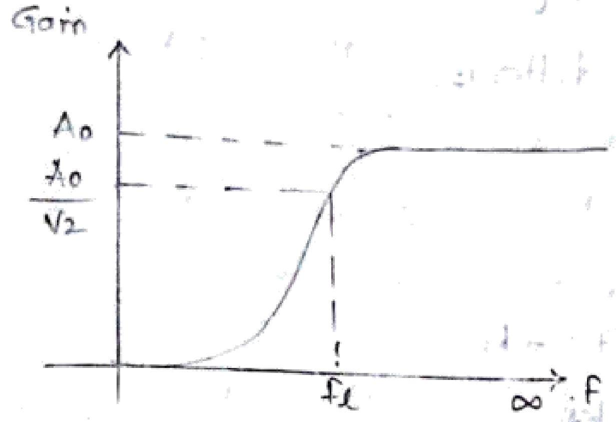
$$\frac{V_0(f)}{V_{in}(f)} = \frac{A_0}{1 + \frac{1}{j2\pi fRC}}$$

If $f = 0$, V_{in} is a dc signal,

$$\frac{V_0(0)}{V_{in}(0)} = 0$$

If $f = \infty$, V_{in} is an AC signal with high frequency.

$$\frac{V_o(\infty)}{V_{in}(\infty)} = A_o = 1 + \frac{R_f}{R_1}$$



At $f = f_l$, $\left|\frac{V_o}{V_{in}}\right| = \frac{A_o}{\sqrt{2}}$

$$\left|\frac{V_o(f)}{V_{in}(f)}\right| = \frac{A_o}{\sqrt{1+\left(\frac{1}{2\pi fRC}\right)^2}}$$

At $f = f_l$ (cut off frequency),

$$\left|\frac{V_o(f_l)}{V_{in}(f_l)}\right| = \frac{A_o}{\sqrt{1+\frac{1}{2\pi f_l RC}}}$$

$$\frac{1}{\sqrt{2}} \cdot \cancel{A_o} = \frac{\cancel{A_o}}{\sqrt{1+(2\pi f_l RC)^{-1}}}$$

$$f_l = \frac{1}{2\pi RC}$$

The maximum gain of the filter is $A_o = 1 + \frac{R_f}{R_1}$ and the cutoff frequency is $f_l = \frac{1}{2\pi RC}$.

- Design a butterworth I^{st} order low pass filter with a maximum gain of 5, with a cutoff frequency 10 kHz and plot the frequency response.

The maximum gain of the filter is,

$$A = 1 + \frac{R_f}{R_1}$$

$$5 = 1 + \frac{R_f}{R_1}$$

$$R_f / R_1 = 4 \quad \Rightarrow R_f = 4R_1$$

Let $R_1 = 1\,k\Omega$, then $R_f = 4\,k\Omega$

from the expression for ~~time~~ cutoff frequency,

$$f_h = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_h}$$

$$= \frac{1}{2\pi \times 10 \times 1000}$$

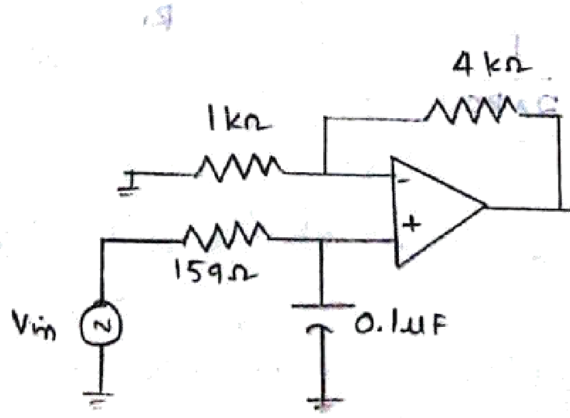
$$= \frac{0.0159}{1000}$$

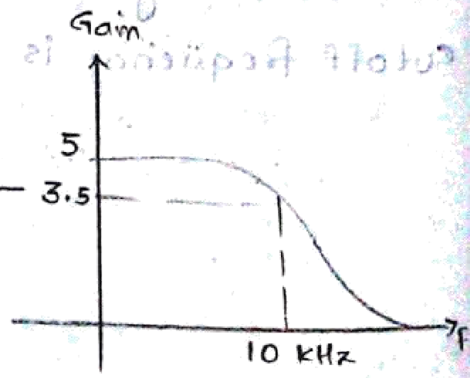
Let $C = 0.1\mu F$,

$$R = \frac{0.0159}{C}$$

$$= \frac{0.0159}{0.1 \times 10^{-6} \times 10^{3}} = 159\,k\Omega \times 10^{-3}$$

$$= 159\Omega$$





- Design a butterworth first order active high pass filter, with a maximum gain 9, cutoff frequency 1kHz.

The maximum gain of the filter is,

$$A_0 = 1 + \frac{R_f}{R_1}$$

$$9 = 1 + \frac{R_f}{R_1}$$

$$R_f = 8R_1$$

Let $R_1 = 1k\Omega$, then $R_f = 8k\Omega$

from the expression for cutoff frequency,

$$f_l = \frac{1}{2\pi RC}$$

$$1kHz = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi \times 10^3}$$

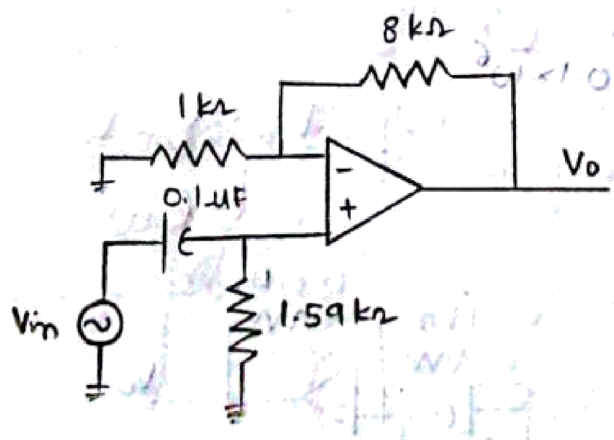
$$= 0.1591 \times 10^{-3}$$

Let $C = 0.1\mu F$,

$$R = \frac{1}{2\pi \times 10^3 \times 0.1 \times 10^{-6}}$$

$$= 1591.54\,\Omega$$

$$= 1.59\,k\Omega$$



- Design butterworth active low pass and high pass filters with maximum gain 20 dB and with cutoff frequency 100kHz and plot the frequency response

The maximum gain,

$$A = 1 + \frac{R_F}{R_1}$$

$$20\log\left[1+\frac{R_F}{R_1}\right] = 20$$

$$1+\frac{R_F}{R_1} = 10$$

$$\frac{R_F}{R_1} = 9$$

$$R_F = 9R_1$$

Let $R_1 = 1\,k\Omega$, $R_F = 9\,k\Omega$

- As the cutoff frequency is 100kHz,

$$f_c = \frac{1}{2\pi RC}$$

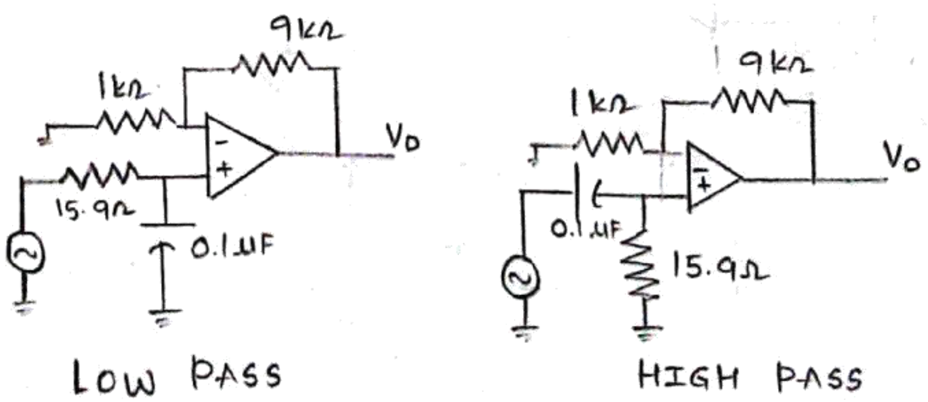
$$10^3 \times 100 = \frac{1}{2\pi RC}$$

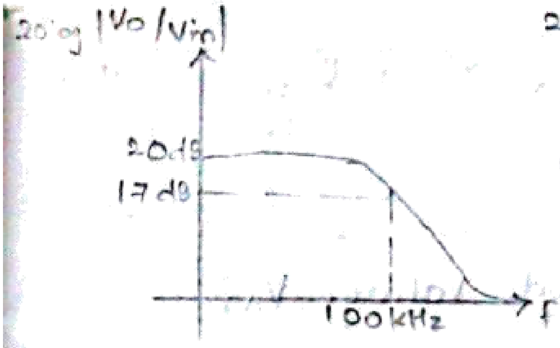
$$R = \frac{1}{2\pi \times 10^5 \times C}$$

Let $C = 0.1\mu F$

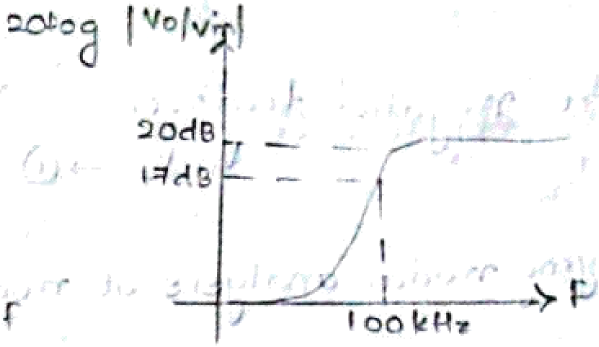
$$R = \frac{1}{2\pi \times 10^5 \times 0.1 \times 10^{-6}}$$

$$= 15.9\Omega$$



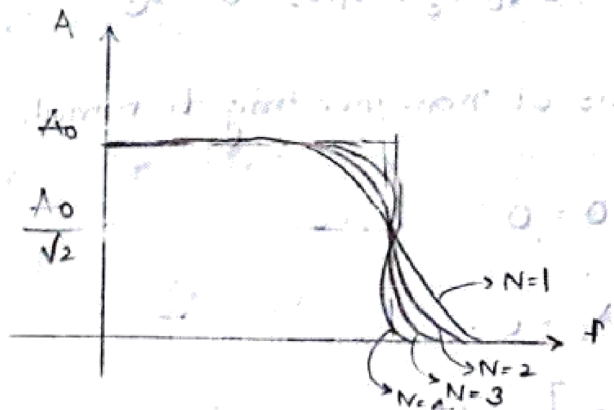


LOW PASS



HIGH PASS

ROLL OFF AND ORDER OF SYSTEM:

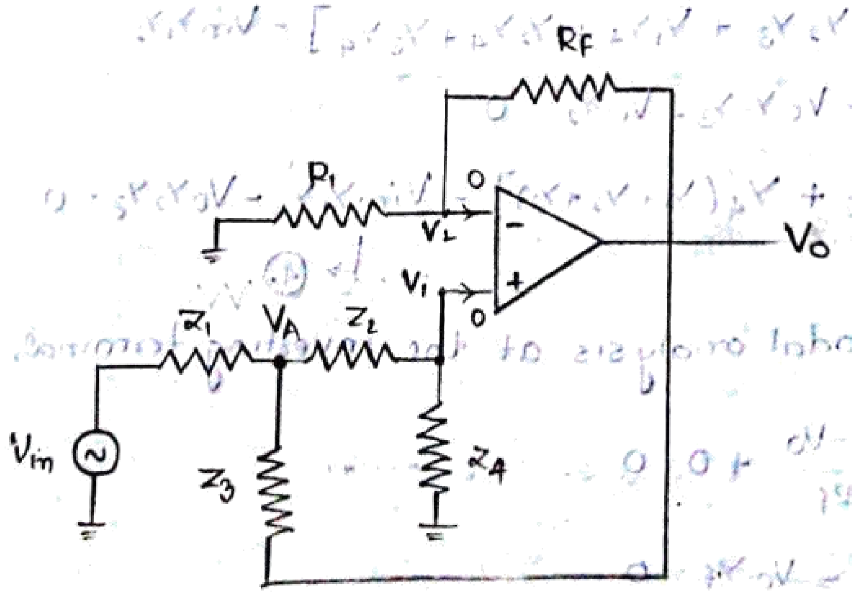


As order of the filter (N) increases, the roll off will be increased. for ideal filter, roll off is infinite

To increase rolloff, we have to increase the order

↳ storage elements in the circuit

SALLEN-KEY MODEL:



Due to negative feedback,

$$V_1 = V_2 \rightarrow (1)$$

Applying nodal analysis at node voltage V_A,

$$\frac{V_A - V_{in}}{Z_1} + \frac{V_A - V_o}{Z_3} + \frac{V_A - V_1}{Z_2} = 0$$

$$V_A\left[Y_1 + Y_2 + Y_3\right] - V_{in} \cdot Y_1 - V_o \cdot Y_3 - V_1 \cdot Y_2 = 0 \rightarrow (2)$$

Applying nodal analysis at non-inverting terminal,

$$\frac{V_1 - V_A}{Z_2} + \frac{V_1 - 0}{Z_4} + 0 = 0$$

$$V_1\left[Y_2 + Y_4\right] - V_A \cdot Y_2 = 0.$$

$$V_A = V_1\left[1 + \frac{Y_4}{Y_2}\right] \rightarrow (3)$$

Substituting (3) in (2),

$$V_1\left[1 + \frac{Y_4}{Y_2}\right]\left[Y_1 + Y_2 + Y_3\right] - V_{in}Y_1 - V_o \cdot Y_3 - V_1 \cdot Y_2 = 0$$

$$V_1(Y_4 + Y_2)(Y_1 + Y_2 + Y_3) - V_{in}Y_1Y_2 - V_oY_2Y_3 - V_1 \cdot Y_2^2 = 0$$

$$V_1\left[Y_1Y_2 + \cancel{Y_2^2} + Y_2Y_3 + Y_1Y_4 + Y_2Y_4 + Y_3Y_4\right] - V_{in}Y_1Y_2$$
$$- V_oY_2Y_3 - \cancel{V_1 \cdot Y_2^2} = 0$$

$$V_1\left[Y_1Y_2 + Y_2Y_3 + Y_4(Y_1 + Y_2 + Y_3)\right] - V_{in} \cdot Y_1Y_2 - V_oY_2Y_3 = 0$$

$$\rightarrow (4)$$

Applying the nodal analysis at the inverting terminal,

$$\frac{V_2 - 0}{R_1} + \frac{V_2 - V_o}{R_f} + 0 = 0$$

$$V_2\left[\cancel{Y_1 + Y_f] - V_o \cdot Y_f}\right. = 0$$

$$V_2\left[\frac{R_1+R_f}{R_1R_f}\right] = \frac{V_o}{R_f}$$

$$V_o = V_2\left[1+\frac{R_f}{R_1}\right]$$

$$= V_2\,[A_o]$$

$$A_o = \frac{V_o}{V_2} \Rightarrow V_2 = \frac{V_o}{A_o} \rightarrow (5)$$

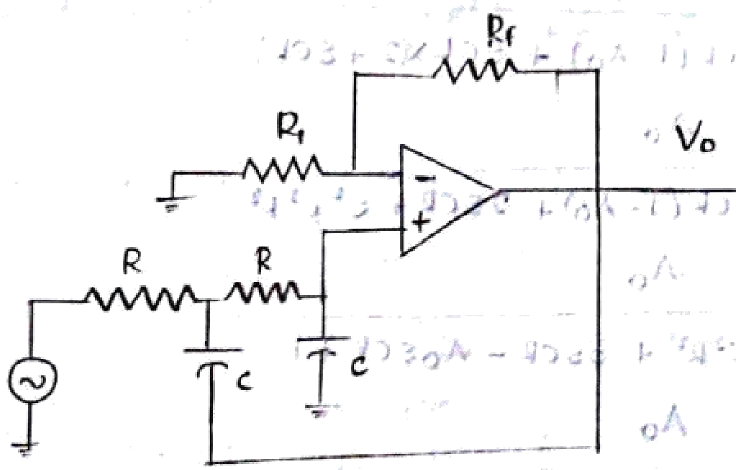
Substituting (5) in (4),

$$\frac{V_o}{A_o}\left[Y_1Y_2 + Y_2Y_3 + Y_4(Y_1+Y_2+Y_3)\right] = V_{in}\,Y_1Y_2 + V_o\,Y_2Y_3$$

$$V_o\left[Y_1Y_2 + (1-A_o)Y_2Y_3 + Y_4(Y_1+Y_2+Y_3)\right] = V_{in}\cdot Y_1Y_2A_o$$

$$\frac{V_o}{V_{in}} = \frac{Y_1Y_2A_o}{Y_1Y_2+(1-A_o)Y_2Y_3+Y_4(Y_1+Y_2+Y_3)}$$

Butterworth ~~first~~ find order Active Low Pass filter:



By comparing the above circuit with Sallen key model,

$Z_1 = R$

$Z_2 = R$

$Z_3 = 1/sC$

$Z_4 = 1/sC$

$$Y_1 = 1/R$$
$$Y_2 = 1/R$$
$$Y_3 = sC$$
$$Y_4 = sC$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1/R \cdot \frac{1}{R} \cdot A_0}{\frac{1}{R^2} + \frac{sC}{R}(1-A_0) + \frac{sC}{R} + \frac{sC}{R} + s^2C^2}$$

$$= \frac{\frac{A_0}{R^2}}{\frac{1}{R^2} + \frac{sC}{R}(1-A_0) + \frac{2sC}{R} + s^2C^2}$$

$$= \frac{A_0/\cancel{R^2}}{\frac{1}{\cancel{R^2}}\left[1 + sCR(1-A_0) + sCR^2\left(\frac{2}{R} + sC\right)\right]}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{A_0}{1 + sCR(1-A_0) + sCR \times (2 + sCR)}$$

$$= \frac{A_0}{1 + sCR(1-A_0) + 2sCR + s^2C^2R^2}$$

$$= \frac{A_0}{s^2C^2R^2 + 3sCR - A_0 sCR + 1}$$

$$= \frac{A_0}{(sCR)^2 + sCR(3-A_0) + 1}$$

In frequency domain,

$$\frac{V_o(f)}{V_{in}(f)} = \frac{A_0}{(j2\pi fCR)^2 + (j\omega RC)(3-A_0) + 1}$$

$$= \frac{A_0}{1 - (2\pi fRC)^2 + j2\pi fRC(3-A_0)}$$

$$\left|\frac{V_o(f)}{V_{in}(f)}\right| = \frac{A_o}{\sqrt{(1-(2\pi fRC)^2)^2 + (2\pi fRC(3-A_o))^2}}$$

Let $f_h = \frac{1}{2\pi RC}$,

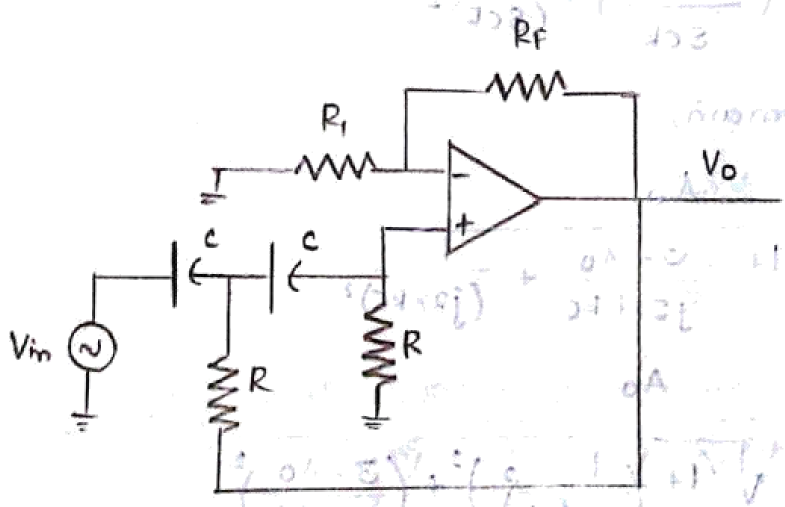
$$\frac{V_o(f)}{V_{in}(f)} = \frac{A_o}{\sqrt{(1-(\cancel{2\pi f_h RC} f/f_h)^2)^2 + (3-A_o)^2 (f/f_h)^2}}$$

$$= \frac{A_o}{\sqrt{\left[1-(f/f_h)^2\right]^2 + \left[(3-A_o)^2 (f/f_h)^2\right]^2}}$$

At $f=0$, $\frac{V_o(f)}{V_{in}(f)} = A_o$

At $f=\infty$, $\frac{V_o(f)}{V_{in}(f)} = 0.$

BUTTERWORTH IInd ORDER ACTIVE HIGH PASS FILTER



Comparing the above with the Sallen key model,

$Z_1 = 1/sC$ $\quad Z_3 = R$

$Z_2 = 1/sC$ $\quad Z_4 = R$

24-09-15

Due to negative feedback,

$$V_1 = V_2$$

$$Y_1 = sC$$

$$Y_2 = sC$$

$$Y_3 = 1/R$$

$$Y_4 = 1/R$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{(sC)^2 A_o}{(sC)^2 + (1-A_o)\frac{sC}{R} + \frac{1}{R}\left(sC + sC + \frac{1}{R}\right)}$$

$$= \frac{(sC)^2 A_o}{(sC)^2 + (1-A_o)\frac{sC}{R} + \frac{1}{R}\left(2sC + \frac{1}{R}\right)}$$

$$= \frac{(sC)^2 A_o}{(sC)^2 + (3-A_o)\frac{sC}{R} + \frac{1}{R^2}}$$

$$= \frac{A_o}{1 + \left(\frac{3-A_o}{sCR}\right) + \frac{1}{(sCR)^2}}$$

In frequency domain,

$$\frac{V_o(f)}{V_{in}(f)} = \frac{A_o}{1 + \frac{3-A_o}{j2\pi fRC} + \frac{1}{(j2\pi RC)^2}}$$

$$\left|\frac{V_o(f)}{V_{in}(f)}\right| = \frac{A_o}{\sqrt{\left(1 + \left(\frac{1}{j2\pi RC}\right)^2\right)^2 + \left(\frac{3-A_o}{2\pi fRC}\right)^2}}$$

The quantity, $f_B = \frac{1}{2\pi RC}$

$$= \frac{A_o}{\sqrt{1 + \left(-\frac{1}{\pi RC}\right) + \left(\frac{3-A_o}{2\pi fRC}\right)^2}}$$

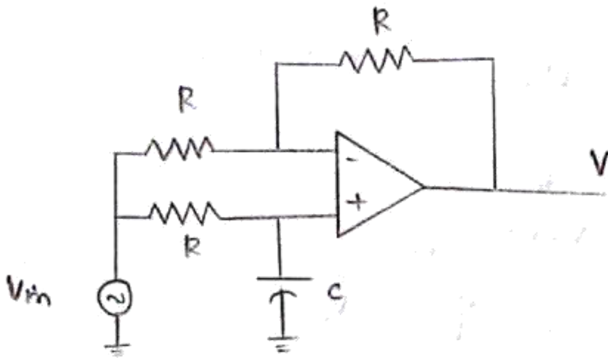
$$= \frac{-A_o}{1 - \frac{j(3-A_o)}{2\pi f RC} + \frac{-1}{(2\pi RC)^2}}$$

$$\left|\frac{V_o(f)}{V_{in}(f)}\right| = \frac{A_o}{\sqrt{\left(1 - \frac{1}{(2\pi RC)^2}\right)^2 + \left(\frac{3-A_o}{2\pi f RC}\right)^2}}$$

Let $f_l = \frac{1}{2\pi RC}$

$$\left|\frac{V_o(f)}{V_{in}(f)}\right| = \frac{A_o}{\sqrt{\left[1 - \left(\frac{f_l}{f}\right)^2\right]^2 + \left[3 - A_o \cdot \left(\frac{f_l}{f}\right)\right]^2}}$$

ALL PASS FILTER:



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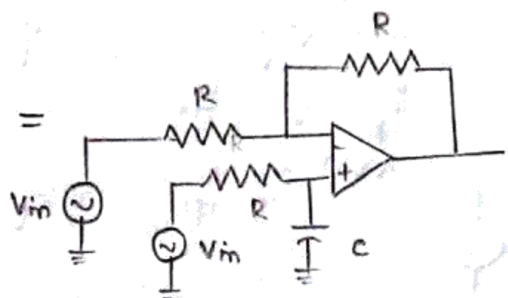
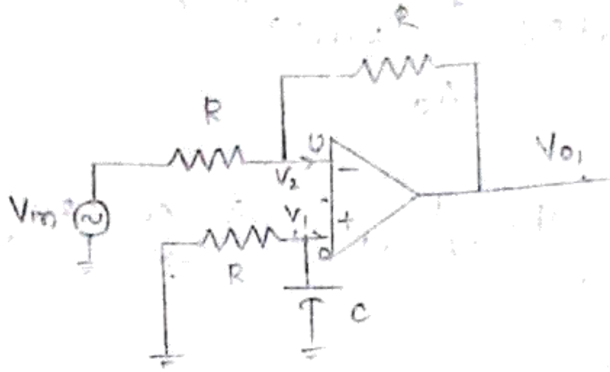


According to the superposition theorem,
In an LTI system consisting of more one source, the total response at one element is equal to algebraic sum of individual responses.

Two cases are obtained,
(i) Output V_{o1}
(ii) Output V_{o2}

Case (i):



for negative feed back,

$$V_1 = V_2$$

Since $V_1 = 0$,

$$V_1 = V_2 = 0$$

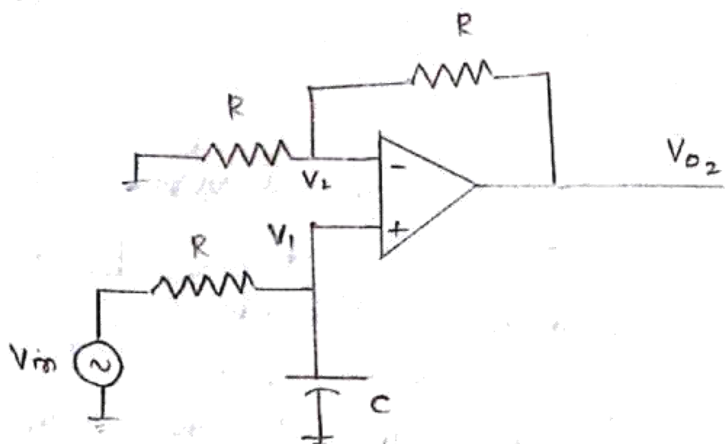
Applying nodal analysis at inverting terminal,

$$\frac{V_2 - V_{in}}{\cancel{R}} + \frac{V_2 - V_{o_1}}{\cancel{R}} = 0$$

$\because$ $V_2 = 0$

$$V_{o_1} = -V_{in}$$

case (ii)



Applying voltage divider rule at V_1,

$$V_1 = V_{in}(s) \cdot \frac{1/sC}{1/sC + R}$$

$$= \frac{V_{in}(s)}{1+sCR}$$

for negative feedback,

$$V_1 = V_2 .$$

Applying the nodal analysis at inverting terminal,

$$\frac{V_2 - 0}{R} + \frac{V_2 - V_{O_2}}{R} = 0$$

$$V_{O_2} = RV_2$$

$$V_{O_2} = \frac{2}{1+sCR} \cdot V_{in}(s)$$

Total Response,

$$V_O = V_{O_1} + V_{O_2}$$

$$= -V_{in}(s) + \frac{2}{1+sCR} \cdot V_{in}(s)$$

$$\frac{V_O(s)}{V_{in}(s)} = -1 + \frac{2}{1+sCR}$$

$$= \frac{-1 - sCR + 2}{1+sCR}$$

$$= \frac{1 - sCR}{1+sCR}$$

In frequency domain,

$$\frac{V_O(f)}{V_{in}(f)} = \frac{1 - j2\pi f RC}{1 + j2\pi RC}$$

$$\left|\frac{V_O(f)}{V_{in}(f)}\right| = \frac{\cancel{\sqrt{1+(2\pi f RC)^2}}}{\cancel{\sqrt{1+(2\pi f RC)^2}}}$$

$$= 1$$

$$\angle\frac{V_O(f)}{V_{in}(f)} = \tan^{-1}(-2\pi f RC) - \tan^{-1}(2\pi f RC)$$

$$\tan^{-1}(-\theta) = -\tan^{-1}(\theta)$$

$$= -\tan^{-1}(2\pi f RC) - \tan^{-1}(2\pi RC)$$

$$= -2\tan^{-1}(2\pi f RC).$$

$f \to 0$ (d.c signal) $\rightarrow \angle\frac{V_O(f)}{V_{in}(f)} = 0$

$f \to \infty$ (a.c. signal) $\rightarrow \angle\frac{V_O(f)}{V_{in}(f)} = -180^\circ$

The gain magnitude is 1, i.e, independent of the frequency, so, output is equal to input. But, a phase change exists between the input and output based on the input signal frequency.
It can generate −180° to 0° phase shift between the output and input. i.e. input leads the output.

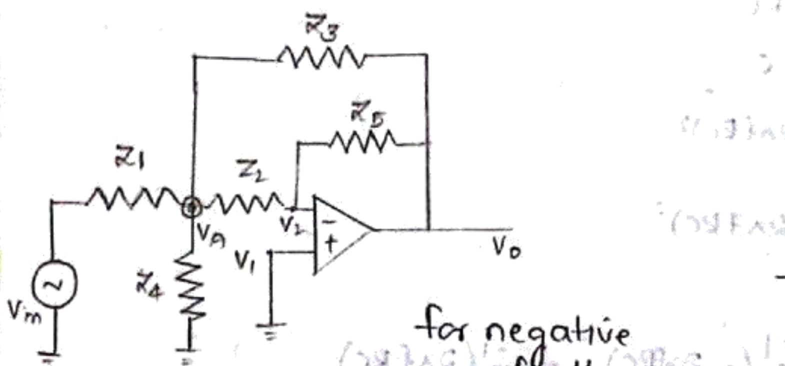
BAND PASS FILTER:

28-09-15

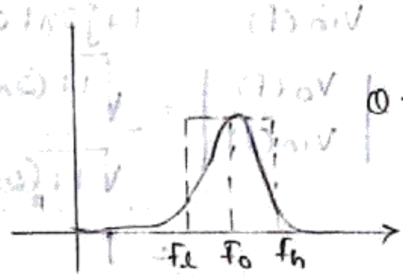
- → Narrow Band Pass (B.W is less) $Q > 10$
- → Wide Band Pass (B.W is more) $Q < 10$

$$\underset{\text{Quality factor}}{Q} = \frac{f_0}{B.W} = \frac{f_0}{f_h - f_L} \quad \text{where } \underset{\text{Centre frequency}}{f_0} = \sqrt{f_h f_L}$$

NARROW BAND PASS FILTER



for negative feedback, $V_1 = V_2$



$Q > 10$

Nodal analysis at mode voltage V_A,

$$\frac{V_A - V_{in}}{Z_1} + \frac{V_A - V_2}{Z_2} + \frac{V_A - 0}{Z_4} + \frac{V_A - V_0}{Z_3} = 0$$

But $V_2 = 0$,

$$V_A\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4}\right] - \frac{V_{in}}{Z_1} - \frac{V_0}{Z_3} = 0$$

$$V_A[Y_1+Y_2+Y_3+Y_4] = V_{in}Y_1 + V_o \cdot Y_3 \rightarrow (1)$$

Applying nodal analysis at inverting node,

$$\frac{V_2 - V_A}{Z_2} + \frac{V_2 - V_o}{Z_5} + 0 = 0$$

But $V_2 = 0$,

$$-V_A \cdot Y_2 = V_o Y_5$$

$$V_A = -\frac{Y_5}{Y_2} \cdot V_o \rightarrow (2)$$

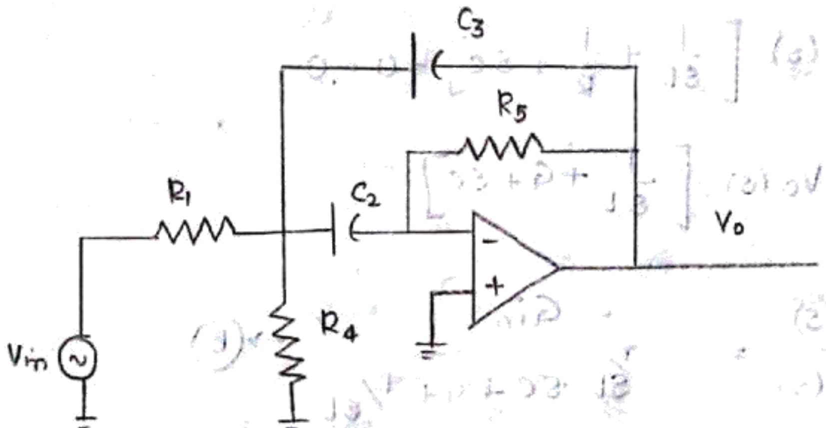
from (1) and (2),

$$-\frac{Y_5}{Y_2} \cdot V_o [Y_1+Y_2+Y_3+Y_4] = V_{in} Y_1 + V_o Y_3$$

$$-V_o Y_5 [Y_1+Y_2+Y_3+Y_4] = V_{in}(Y_1 Y_2) + V_o (Y_3 Y_2)$$

$$-V_o [Y_2 Y_3 + Y_5(Y_1+Y_2+Y_3+Y_4)] = V_{in}(Y_1 Y_2)$$

$$\frac{V_o}{V_{in}} = \frac{-Y_1 Y_2}{Y_2 Y_3 + Y_5 (Y_1+Y_2+Y_3+Y_4)}$$



$Z_1 = R_1$ $\quad Z_2 = 1/sC_2$ $\quad Z_3 = 1/sC_3$ $\quad Z_4 = R_4$ $\quad Z_5 = R_5$

$Y_1 = G_1$ $\quad Y_2 = sC_2$ $\quad Y_3 = sC_3$ $\quad Y_4 = G_4$ $\quad Y_5 = G_5$

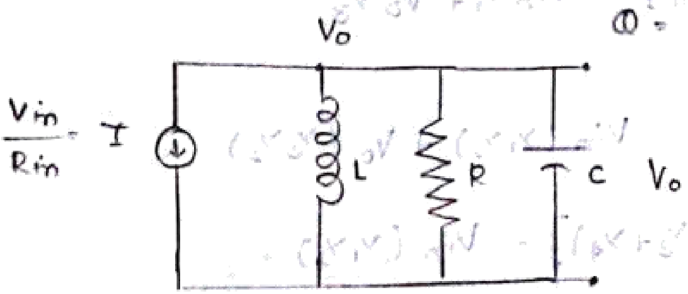
$$\frac{V_o}{V_{in}} = \frac{-(G_1 \cdot sC_2)}{(sC_2)(sC_3) + G_5[G_1 + sC_2 + sC_3 + G_4]}$$

$$= \frac{-G_1 \cdot sC_2}{sC_2 \cdot sC_3 + G_1G_5 + sC_2 \cdot G_5 + G_5 \cdot sC_3 + G_4G_5}$$

Dividing with sC_2,

$$= \frac{-G_1 \cdot \cancel{sC_2}}{sC_3 + G_5\left[1 + \frac{C_3}{C_2}\right] + \frac{G_5}{sC_2}[G_1 + G_4]} \rightarrow (A)$$

RLC Parallel Circuit:



$Q = \frac{I_L}{I_R}$ or $\frac{I_C}{I_R}$ (Parallel Resonance)

$Q = \frac{V_L}{V_R}$ or $\frac{V_C}{V_R}$ (Series Resonance)

Applying Nodal Analysis,

$$I + \frac{V_o(s)}{sL} + \frac{V_o(s)}{R} + \frac{V_o(s)}{1/sC} + 0 = 0 \quad \curvearrowright \text{Open circuit}$$

$$\frac{V_{in}}{R_{in}} + V_o(s)\left[\frac{1}{sL} + \frac{1}{R} + sC\right] + 0 = 0$$

$$\frac{V_{in}}{R_{in}} = -V_o(s)\left[\frac{1}{sL} + G + sC\right]$$

$$\frac{V_{in}}{V_o(s)} \quad \frac{V_o(s)}{V_{in}(s)} = \frac{-G_{in}}{\cancel{sL} \cdot sC + G + 1/sL} \rightarrow (B)$$

Resonance (central): $\omega_o = \frac{1}{\sqrt{LC}}$

Max. Gain $\left|\frac{V_o}{V_{in}}\right| = \frac{-G_{in}}{G}$

Quality Factor. $Q_0 = \frac{R}{\omega_0 L} = \omega_0 RC$

Bandwidth $= \frac{f_0}{Q}$ $[\because Q = f_0/B.W]$.

On comparision with Ⓐ, and with Ⓑ, (the transfer function of RLC Parallel circuit) (transfer function of IVBPF) we get.

$G_{in} = G_1$

$G = G_5 \left[1 + \frac{C_3}{C_2}\right]$

$C = C_3$

$L = \frac{C_2}{G_5 [G_1 + G_4]}$

Resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{\frac{C_2}{G_5(G_1+G_4)} \cdot C_3}} = \sqrt{\frac{G_5(G_1+G_4)}{C_2 C_3}}$$

Maximum Gain, $A_0 = \left|\frac{V_0}{V_{in}}\right|$

$$= -\frac{G_{in}}{G} = \frac{-G_1}{G_5\left[1+\frac{C_3}{C_2}\right]} = \frac{-G_1 C_2}{G_5(C_2+C_3)}$$

Quality factor, $Q = \frac{R}{\omega_0 L} = \frac{1}{\omega_0 G L}$

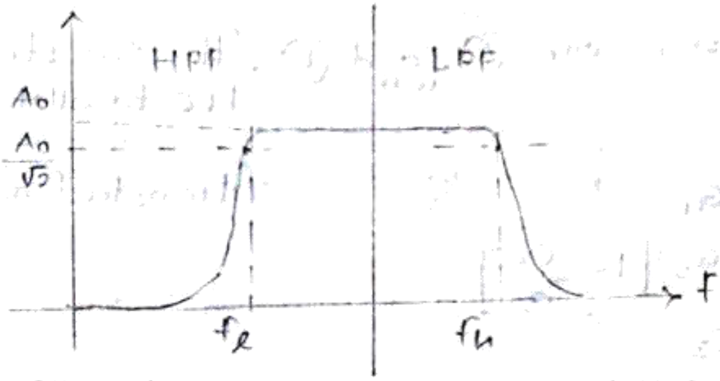
$$= \frac{1}{\sqrt{\frac{G_5(G_1+G_4)}{C_2 C_3}} \cdot \cancel{G_5}\left[\frac{C_2+C_3}{\cancel{C_2}}\right] \cdot \frac{\cancel{C_2}}{\cancel{G_5}[G_1+G_4]}}$$

$$= \frac{\sqrt{(G_1+G_4)(C_2 C_3)}}{\sqrt{G_5}\,(C_2+C_3)}$$

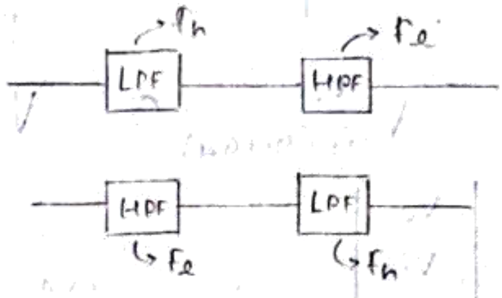
Bandwidth, $= \frac{f_0}{Q} = \frac{\omega_0}{2\pi Q}$

29-09-15

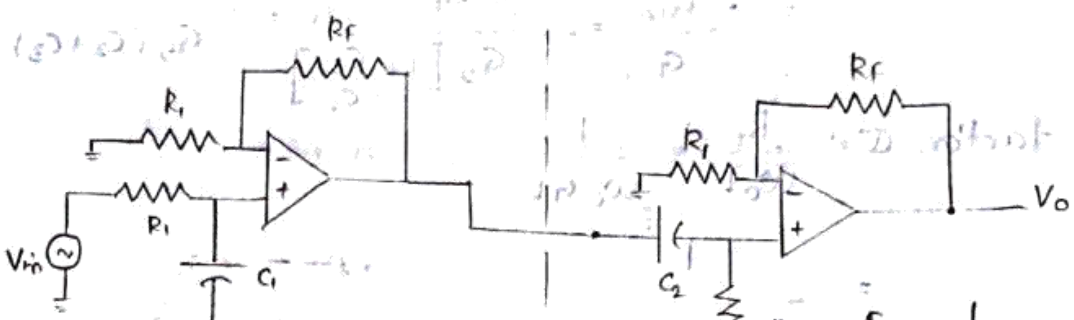
WIDE BAND PASS FILTER:



By the cascade connection of LPF and HPF, we can design the band pass filter.
But the condition is HPF cutoff frequency (f_L) should be lesser than LPF cutoff frequency.



$f_l < f_h$



$f_h = \frac{1}{2\pi R_1 C_1}$

$f_l = \frac{1}{2\pi R_2 C_2}$

Condition: $f_l < f_h$

$$\frac{1}{2\pi R_2 C_2} < \frac{1}{2\pi R_1 C_1}$$

$$R_1 C_1 < R_2 C_2$$

R_1C_1 ↓ LPF, R_2C_2 ↳ HPF

$$\frac{V_o}{V_{in}} = \frac{V_o}{V_{o1}} \times \frac{V_{o1}}{V_{in}}$$

$$\left|\frac{V_o}{V_{o1}}\right| \left|\frac{V_{o1}}{V_{in}}\right|$$

$$\frac{V_o}{V_{in}} = \frac{V_o}{V_{o1}} \cdot \frac{V_{o1}}{V_{in}}$$

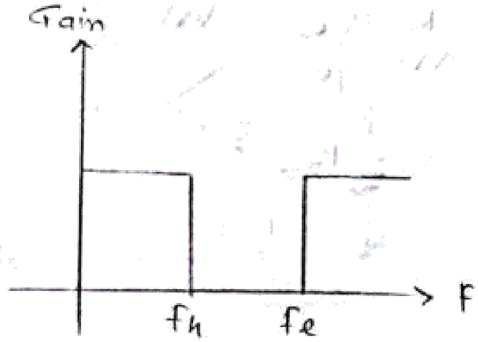
$$\left|\frac{V_o}{V_{in}}\right| = \underbrace{\left|\frac{V_o}{V_{o1}}\right|}_{HPF} \underbrace{\left|\frac{V_{o1}}{V_{in}}\right|}_{LPF}$$

$$\left|\frac{V_o}{V_{in}}\right| = \frac{A_o}{\sqrt{1+(f_l/f)^2}} \cdot \frac{A_o}{\sqrt{1+(f/f_h)^2}}$$

$$= \frac{A_o^2}{\sqrt{\left(1+\left(\frac{f_l}{f}\right)^2\right)\left(1+\left(f/f_h\right)^2\right)}}$$

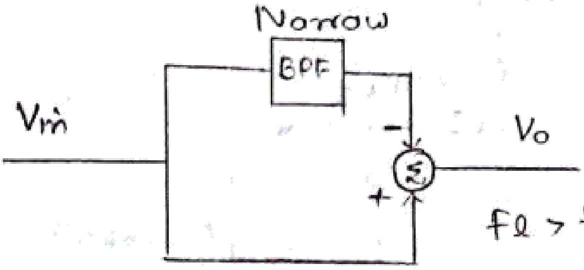
(Max gain $= A_o^2$
$A_o = 1 + \frac{R_f}{R_1}$)

BAND REJECT FILTER:



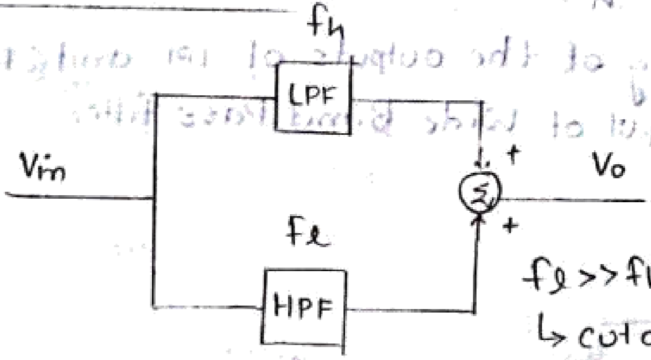
$f_e > f_h$

For Narrow BRF,



$f_e > f_h$

for ~~Narro~~ Wide BRF,



$f_e >> f_h$

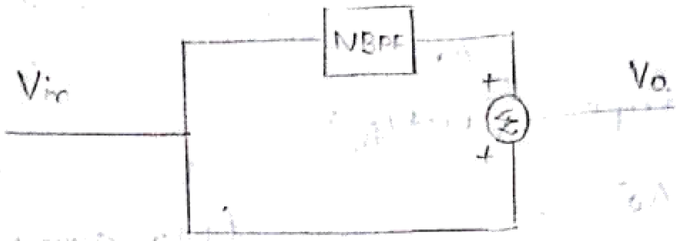
→ cutoff freq. of LPF

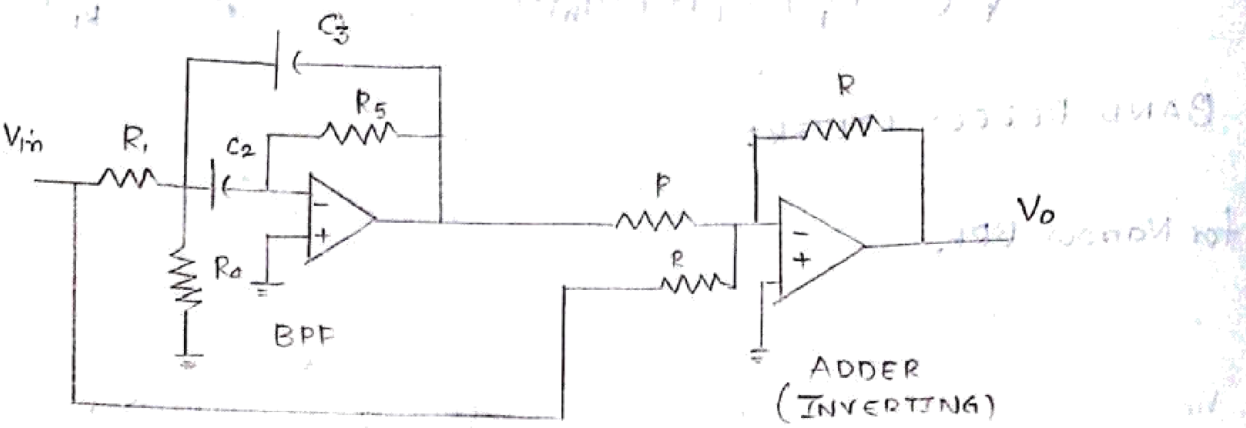
↳ cutoff freq. of HPF

Narrow Band Reject filter:

For this $f_e > f_h$, but f_e is closer to f_h. To design this, we are using the Band pass filter, i.e. by subtracting the band pass filter output from the input, we can design the band reject filter.
But in this, the maximum gain of Band Pass Filter should be '1'.

But NBF filter output is negative (Gain).
So, instead of subtracting from the input, we are adding the NBF output to the input.

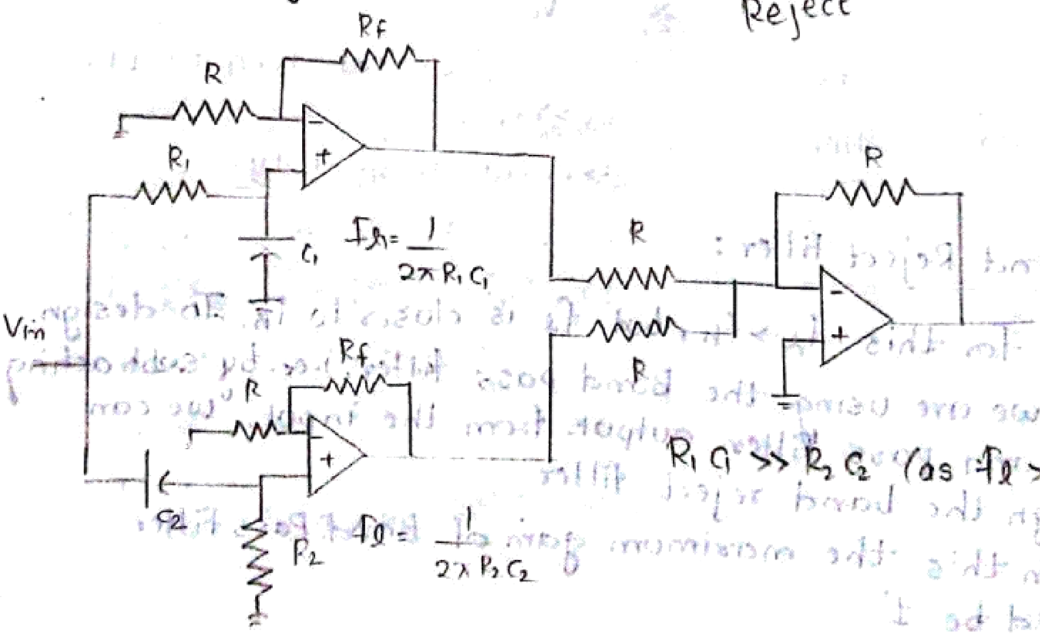




WIDE BAND ~~PASS~~ REJECT FILTER:

In this $f_l >> f_h$.

By adding of the outputs of LPF and HPF, we can get the output of Wide Band Pass filter.
Reject



$R_1 C_1 >> R_2 C_2$ (as $f_l >> f_h$)